Technical Comments

Comment on "An Investigation of the Acceleration Induced Burning Rate Increase of Nonmetalized Composite Propellants"

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A RECENT study by Sturm and Reichenbach[1] extends Fenn's Phalanx flame model to acceleration environments. The mechanism for this extension is the hypothesis that the Phalanx flame consumes the AP/binder interface at a rate substantially greater than the mass burning rate thereby freeing AP particles before their consumption. In a null field the "freed" AP particle literally blows itself away from the burning surface. However, in an acceleration field with a component normal to and into the surface, the body force will retain some of the freed AP particles on the surface. It is hypothesised by Ref. 1 that this increases the energy flow to the burning surface thereby augmenting the burning rate. The authors work out the implications of this model in some detail and then compare the theoretical predictions with strand data for three bimodal PBAN/AP propellants with the same O/F ratio. The agreement is generally quite good.

A particularly interesting consequence of the theory is that the pressured dependence of burning rate $\partial \ln r/\partial \ln p$ varies with both pressure and acceleration and can, at sufficiently high pressure and acceleration, exceed unity. This is a marked departure from the null field Phalanx flame model and implies that a rocket motor can be stable at rest, but become unstable (steady state criteria) at high acceleration. These theoretical consequences are not mentioned in Ref. 1. The purpose of this Note is to work out $\partial \ln r/\partial \ln p$, present limited numerical results from the data of Ref. 1, comment on the model, and recall data in the open literature concerning the generality of the model as stated.

The equation for burning rate ratio derived by Ref. 1 is† $r/r_0 = (1 - \eta f J)^{-1}$. Therefore

$$\partial(r/r_0)/\partial p = (1 - \eta f J)^{-2}[\eta(f\partial J/\partial p + J\partial f/\partial p)] \quad (1)$$

However, $f = Kp^{1/m}$ (K, m = const),‡ $J = J(d_{pc})$, and $d_{pc} = K_1(r/Gp)^{1/2}$. Thus, $\partial f/\partial p = f/(mp)$ and $\partial J/\partial p = J'd_{pc}(r^{-1}\partial r/\partial p - p^{-1})/2$. Some manipulation then yields ($\partial \ln r_0/\partial \ln p = n_0$)

$$\frac{\partial \ln r}{\partial \ln p} = \left[n_0 + \frac{\eta f(J/m - d_{pc}J'/2)}{1 - \eta f J}\right] \times \left[1 - \frac{\eta f J' d_{pc}/2}{1 - \eta f J}\right]^{-1} \quad (2)$$

Examination of these equations shows that pressure dependency arises from the energy released by a particle to the surface (represented by J/m) and conditions governing the amount of particulate matter retained at the surface (represented by $d_{pc}J'/2$). Note that since $J' \leq 0$, these two effects are additive.

Investigation of Eq. (2) at the acceleration extremes shows that

$$\lim_{G\to 0} \partial \ln r/\partial \ln p = n_0 \quad (3)$$

and

$$\lim_{G\to \infty} \partial \ln r/\partial \ln p = n_0 + [(r/r_0)_\infty - 1]/m \quad (4)$$

Equation (3) shows that the theory gives the correct lower limit. Equation (4) shows that $(\partial \ln r/\partial \ln p)_\infty > n_0$. Moreover, since $(r/r_0)_\infty$ increases with pressure, $(\partial \ln r/\partial \ln p)_\infty$ also increases with pressure and will exceed unity at some finite pressure. This implies that a stable static motor may become unstable (steady-state criteria) in an acceleration environment. These consequences are the direct result of the assumption concerning the variation of energy release transferred to the burning surface with pressure ($f = Kp^{1/m}$).

To find conditions defining the stability boundary set $\partial \ln r/\partial \ln p = 1$ in Eq. (2). Some manipulation gives§

$$p_1 = [(m/\eta K J)(1 - n_0)/\{1 + m(1 - n_0)\}]^m \quad (5)$$

while further rearrangement shows that

$$(r/r_0)_1 = 1 + m(1 - n_0) \quad (6)$$

Thus, $\partial \ln r/\partial \ln p$ always exceeds unity at the same burning rate ratio with the same propellant.

Examples of the magnitude of these effects have been computed for propellant P411 using $m = 3$, $n_0 = \frac{1}{3}$, and data supplied in Ref. 1. Figure 1 shows the variation of $\partial \ln r/\partial \ln p$ with acceleration for several pressures. Note that large changes occur at the higher pressures and that $\partial \ln r/\partial \ln p$ can exceed its $G \to \infty$ limiting value. The discontinuity results from the lack of smoothness in the J function at $d_{pc} = 2.5\ \mu$. The figure also shows that $\partial \ln r/\partial \ln p$ increases with pressure when $G > 0$. Figure 2 illustrates the stability boundary and shows that instabilities should occur near 5000 psi for $G > 500g$. Examination of Eqs. (2) and (5) shows that the trends shown in these figures should be exhibited by any propellant.

Equation (4) shows that at high acceleration $\partial \ln r/\partial \ln p > n_0$. However, Wall's data for polysulfide/AP propellants at ultra high accelerations[2] and Anderson's data for a PBAN/AP propellant[3] indicate that $\partial \ln r/\partial \ln p \simeq n_0$ at high accelera-

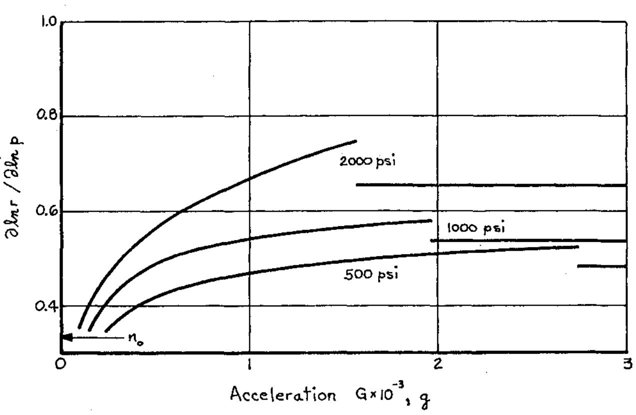

Fig. 1 Effect of acceleration on $\partial \ln r/\partial \ln p$ for P411 propellant.

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† See Nomenclature Ref. 1. Prime denotes differentiation with respect to the argument. A subscript ∞ denotes limiting conditions as $G \to \infty$.

‡ Generalizing the expression given by Ref. 1 where $m = 3$.

§ Subscript 1 denotes conditions where $\partial \ln r/\partial \ln p = 1$.

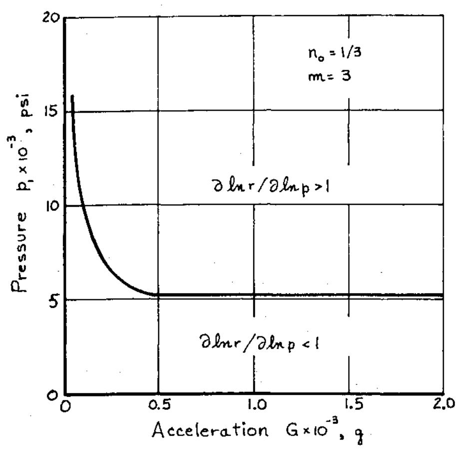

Fig. 2 Stability boundary for propellant P411.

tions. This is impossible within the present structure of the Sturm-Reichenbach theory. However, with the modification in the function f introduced herein this can be achieved if $m = \infty$. Although this guarantees that $(\partial \ln r/\partial \ln p)_\infty = n_0$, $\partial \ln r/\partial \ln p$ will still exceed n_0 for intermediate accelerations. Figure 3 illustrates $\partial \ln r/\partial \ln p$ for propellant P411 with $m = 3$ and $m = \infty$. The figure shows that with $m = \infty$ $\partial \ln r/\partial \ln p$ still attains a maximum value substantially greater than n_0. Note that for accelerations below $200g$, the dependence on m essentially vanishes.

The implications concerning $\partial \ln r/\partial \ln p$ stem from both the criteria determining d_{pc} and the assumption relating to the function f. The former appears to be on a sound physical basis. However, the latter is justified by recourse to null field theories. Since in an acceleration field this effect is determined by phenomena occurring between the "free" AP particles and the propellant, a phenomenon that does not occur in either null field theory, this justification seems weak. It is interesting to note that Willoughby and Crowe[4] abandoned the approach taken by Ref. 1 for one dealing directly with energy transport across the particle/surface clearance in order to achieve qualitative agreement regarding pressure dependence. According to the Phalanx flame model, burning rate is controlled by reaction at the AP/binder interface. The mechanism by which energy released by freed AP particles (this means they have been by-passed by the Phalanx flame) increases the Phalanx flame rate is obscure. Moreover, if the mass burning rate were to exceed the Phalanx rate, free AP particles would no longer be created. Therefore, the Phalanx rate appears to represent an upper bound on burning rate that is not included in the model as stated. Finally, data for TPG-3016D propellant (Thiokol designation) presented by Northam[5] show that $r/r_0 \neq 1$ when the acceleration is away from the burning surface—a phenomenon disallowed by the model. However, in Ref. 6, results do show $r/r_0 = 1$ with parallel accelerations.

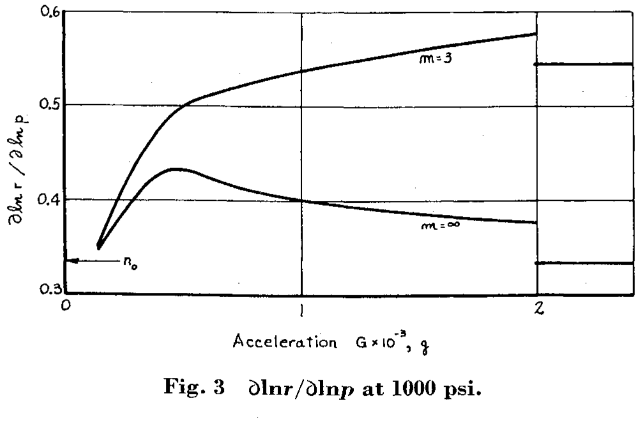

Fig. 3 $\partial \ln r/\partial \ln p$ at 1000 psi.

In summary the work reported here shows that the Sturm-Reichenbach theory leads to $\partial \ln r/\partial \ln p > n_0$ for $G > 0$. However, some data reported in the literature indicate that $\partial \ln r/\partial \ln p \simeq n_0$. In addition, some data on the variation of burning rate with acceleration direction conflicts with the theory as stated. These results suggest that the theory has limited validity. However, since the pressure dependency difficulties can be partially overcome and the theory provides a framework for interpolation/extrapolation with limited data, additional tests of this theory should be encouraged, especially those at high pressure ($\geq$2000 psi) and high acceleration ($\geq 200g$).

References

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Reply by Authors to R. L. Glick

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GLICK has derived some interesting consequences of the theory relating to the pressure and acceleration dependence of $\partial \ln r/\partial \ln p$. In deriving his Eqs. (4, 5, and 6) (from which subsequently were plotted the straight line portions of Fig. 1, and Figs. 2 and 3) Glick made use of the assumption that

$$\lim_{G \to \infty} J' d_{pc} = 0$$

Although this limit is mathematically correct, a zero value of $J'd_{pc}$ is not admissible within the framework of the model as it is postulated. As a result, Glick's Eqs. (4–6) are not considered valid. However, Eq. (2) and its consequences are worthy of note and will be discussed subsequent to the discussion of the term $J'd_{pc}$ and its limiting value from a physical standpoint.

Let us first consider the factor d_{pc}. Although not explicitly stated in the mathematical derivation, there exists on physical grounds a lower limit (greater than zero) for d_{pc}.

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